Remarks by Tracy A. Pierce, Harvard University.

It is well known that every prime of the form $4m + 1$ is the sum of two squares. Using Lucas's form of an odd perfect number (see Problem 211 above) we see that

$$(4m + 1)^{4k+1}n^2 = (x^2 + y^2)[(4m + 1)^{2k}]^2n^2 = (x^2 + y^2)N^2 = X^2 + Y^2,$$

contrary to the proposition as proposed.

229. (March, 1915.) Proposed by WALTER C. EELLS, Whitman College.

If p and q are integers and p is prime and positive, find the condition on q that the equation $p^x = qx$ shall have integral solutions, solve for x, and show that for a special value of p it has two solutions for a certain q, otherwise only one.

I. Solution by Frank Irwin, University of California.

Since p is prime, we must have $x = p^a$, $q = p^b$, where a and b are positive integers or zero (for a). Now $p^{a+b} = qx = p^{p^a}$, so that $b = p^a - a$, and q is necessarily of the form $p^{(p^a-a)}$. This condition is evidently also sufficient.

Given, then, such a q, the exponent $p^a - a$ may be determined; then a, which will give us x, is that number which must be added to this exponent to make it equal to the *next higher* power of p. For no power of p can lie between $p^a - a$ and p^a, since $p^a - a > p^{a-1}$, as may be readily proved, for instance by mathematical induction.

Of the cases that require special investigation, $a = 0$, and $p = 2$ with $a = 1$ or 2, the only one for which, given $p^a - a$, there is more than one solution for a, is the case

$$2^a - a = 1,$$

which has two solutions $a = 0, 1$. There are two solutions of our problem then for the case $p = 2$, $q = 2$, viz., $x = 1, 2$.

II. Solution by the Proposer.

Consider the two functions, $y = p^x$, $y = qx$. For $x = k$ (any integer), $y = p^k$. The slope of line $y = qx$, passing through (k, p^k), is $q = p^k/k$, which is integral if and only if

$$k = p^n \quad (n = 0, 1, 2, 3, \cdots), \text{ } i.\ e., \ q = p^{(p^n-n)}.$$

(If $k < 0$, q is fractional since it is then $= 1/kp^k$).

Substituting this value of q in the given equation, it is easily seen that it is satisfied if and only if

$$x = p^n \quad (n = 0, 1, 2, 3, \cdots).$$

Consider the exponent of p, namely $p^n - n$. We have

$$[p^n - n]_{n=0} = 1, \quad \text{and} \quad [p^n - n]_{n=1} = p - 1.$$

Then $x_1 = p^0$ and $x_2 = p^1$ will be solutions of the given equation if $1 = p - 1$, *i. e.*, if $p = 2$. From the graphs of the exponential function it is easily seen that $y = qx$ can have but one integral intersection if $p \neq 2$, $n_1 \neq 0$, $n_2 \neq 1$.

The equation having two solutions is $2^x = 2x$, of which $x_1 = 1$, $x_2 = 2$.

230. (April, 1915.) Proposed by E. B. ESCOTT, Ann Arbor, Michigan.

Find three numbers such that their sum, the sum of their squares, and the sum of their cubes, shall be a cube.

Note.—W. D. Cairns says this problem, which was proposed in *L'Intermediaire* in 1900, remains unsolved to date, even though it was reprinted in February, 1913.

Remarks by Artemas Martin, Washington, D. C.

The above problem was published in the *Mathematical Visitor*, Vol. I, No. 1 (Erie, Pa., March, 1877), page 6, as No. 9 in a list of "Unsolved Problems." So far as the writer at present knows that was the first publication of the problem and it still remains "unsolved."